

7. EXAMPLE OF A JORDAN NORMAL FORM  
Part I

$$A = \begin{pmatrix} 2 & 2 & 0 \\ -1 & 2 & 2 \\ 0 & 1 & 2 \end{pmatrix} \quad P_A(x) = (x-2)^3 \quad S^{-1} \cdot A \cdot S = J \quad S = (v_1 \ v_2 \ v_3)$$

$$(A - 2 \cdot E) \cdot v_1 = 0 \Leftrightarrow \begin{pmatrix} 0 & 2 & 0 \\ -1 & 0 & 2 \\ 0 & 1 & 0 \end{pmatrix} \cdot v_1 = 0 \Rightarrow v_1 = \begin{pmatrix} 2 \\ 0 \\ 1 \end{pmatrix}$$

$$(A - 2 \cdot E) \cdot v_2 = v_1 \Leftrightarrow \begin{pmatrix} 0 & 2 & 0 \\ -1 & 0 & 2 \\ 0 & 1 & 0 \end{pmatrix} v_2 = \begin{pmatrix} 2 \\ 0 \\ 1 \end{pmatrix} \Rightarrow v_2 = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}$$

$$(A - 2 \cdot E) \cdot v_3 = v_2 \Leftrightarrow \begin{pmatrix} 0 & 2 & 0 \\ -1 & 0 & 2 \\ 0 & 1 & 0 \end{pmatrix} v_3 = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} \Rightarrow v_3 = \begin{pmatrix} -1 \\ 0 \\ 0 \end{pmatrix}$$

$$S = \begin{pmatrix} 2 & 0 & -1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{pmatrix} \quad S^{-1} = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ -1 & 0 & 2 \end{pmatrix} \Rightarrow S^{-1} \cdot A \cdot S = \begin{pmatrix} 2 & 1 & 0 \\ 0 & 2 & 1 \\ 0 & 0 & 2 \end{pmatrix}$$