$$A=\begin{pmatrix}3&0&-1\\0&3&2\\0&0&-1\end{pmatrix} \qquad P_A(X)=\det(X\cdot E-A)=\det\begin{pmatrix}X-3&0&1\\0&X-3&-2\\0&0&X+1\end{pmatrix}=(X-3)^2(X+1)$$

$\lambda=-1$

$$\begin{pmatrix}3&0&-1\\0&3&2\\0&0&-1\end{pmatrix}\begin{pmatrix}v_1\\v_2\\v_3\end{pmatrix}=-1\cdot\begin{pmatrix}v_1\\v_2\\v_3\end{pmatrix} \iff \begin{aligned}3v_1-v_3&=-v_1\\3v_2+2v_3&=-v_2\\-v_3&=-v_3\end{aligned} \iff \begin{aligned}4v_1&=v_3\\-2v_2&=v_3\end{aligned} \Rightarrow v=\begin{pmatrix}1\\-2\\4\end{pmatrix}$$

$\lambda=3$

$$\begin{pmatrix}3&0&-1\\0&3&2\\0&0&-1\end{pmatrix}\begin{pmatrix}v_1\\v_2\\v_3\end{pmatrix}=3\cdot\begin{pmatrix}v_1\\v_2\\v_3\end{pmatrix} \iff \left.\begin{aligned}3v_1-v_3&=3v_1\\3v_2+2v_3&=3v_2\end{aligned}\right\} \Rightarrow v_1, v_2 \text{ arbitrary} \Rightarrow v\in\left\langle\begin{pmatrix}1\\0\\0\end{pmatrix},\begin{pmatrix}0\\1\\0\end{pmatrix}\right\rangle$$

$$-v_3=3v_3 \iff v_3=0$$

$$S=\begin{pmatrix}1&0&1\\0&1&-2\\0&0&4\end{pmatrix} \qquad S^{-1}=\begin{pmatrix}1&0&-\frac{1}{4}\\0&1&\frac{1}{2}\\0&0&\frac{1}{4}\end{pmatrix} \qquad S^{-1}\cdot A\cdot S=\begin{pmatrix}3&0&0\\0&3&0\\0&0&-1\end{pmatrix}$$