2. DIAGONALIZING A 2x2 MATRIX

University of Basel

$$A = \begin{pmatrix} 3 & 2 \\ -1 & 0 \end{pmatrix} \qquad P_A(X) = \det(X \cdot E - A) = \det\begin{pmatrix} X-3 & -2 \\ 1 & X \end{pmatrix} = (X-3)\cdot X - 1 \cdot (-2)$$

$$= X^2 - 3X + 2 = (X-1)(X-2)$$

$\lambda = 1$

$$\begin{pmatrix} 3 & 2 \\ -1 & 0 \end{pmatrix}\begin{pmatrix} v_1 \\ v_2 \end{pmatrix} = 1 \cdot \begin{pmatrix} v_1 \\ v_2 \end{pmatrix} \iff \begin{array}{l} 3v_1 + 2v_2 = v_1 \\ -v_1 = v_2 \end{array} \implies \begin{pmatrix} v_1 \\ v_2 \end{pmatrix} = \begin{pmatrix} 1 \\ -1 \end{pmatrix}, \ \mathrm{Eig}(A,1) = \left\langle \begin{pmatrix} 1 \\ -1 \end{pmatrix} \right\rangle$$

$\lambda = 2$

$$\begin{pmatrix} 3 & 2 \\ -1 & 0 \end{pmatrix}\begin{pmatrix} v_1 \\ v_2 \end{pmatrix} = 2 \cdot \begin{pmatrix} v_1 \\ v_2 \end{pmatrix} \iff \begin{array}{l} 3v_1 + 2v_2 = 2v_1 \\ -v_1 = 2v_2 \end{array} \implies \begin{pmatrix} v_1 \\ v_2 \end{pmatrix} = \begin{pmatrix} 2 \\ -1 \end{pmatrix}, \ \mathrm{Eig}(A,2) = \left\langle \begin{pmatrix} 2 \\ -1 \end{pmatrix} \right\rangle$$

$$S = \begin{pmatrix} 1 & 2 \\ -1 & -1 \end{pmatrix} \qquad S^{-1} = \frac{1}{1}\begin{pmatrix} -1 & -2 \\ 1 & 1 \end{pmatrix} \implies S^{-1} \cdot A \cdot S = \begin{pmatrix} 1 & 0 \\ 0 & 2 \end{pmatrix}$$